and j to two other states. The time intervals and mass-transfer coefficients obtained on samples of the composites analogous to that corresponding to the tomogram in Fig. 1 are considered. It follows from Fig. 3 that: assuming the best material has the least tortuosity, the most acceptable is that in which the layers are characterized by the same properties with respect to moisture migration. In this case, ε is a minimum, and depends only on the ratio L_x/L_y. With a purely diffusional process, $L_y = 0$, and hence ε is not determined. Increase in the difference between $\bar{t}_i^0$ leads to increase in ε. As shown by calculations, moisture migration characterized by the tortuosity is determined basically by the times $\bar{t}_i^0$ and is practically independent of $\bar{\theta}_i$. The results obtained may be extended to any number of layers.

Thus, the tortuosity factor of pores has been estimated as a random parameter of moisture absorption in composites. In the present case, it characterizes the pore space of the material over its whole volume and may be used in mathematical models of moisture transfer.

NOTATION

n, number of layers in material; $\bar{t}_k^0$, mean residence time in k-th state in Fig. 2, taking no account of internal diffusion; L_x, L_y, mass-transfer coefficients along x and y axes, respectively.

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NONSTEADY TRANSFER AND DISPERSIONAL EFFECTS IN HETEROGENEOUS MEDIA

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A single transport equation taking account of the dispersion of effective conductivities and interphase exchange due to relaxation effects, as well as the inhomogeneity of the corresponding fields, is obtained in Laplace transforms. The asymptotes of this equation are considered.

1. The problem of adequate description of heat and mass transfer in heterogeneous and, in particular, granular media has been under intensive study for several decades now. Methods of engineering calculation based on semiempirical models have been proposed, leading to completely satisfactory results in many situations; see the review [1], for example. However, as yet there is no general theory indicating the regions of validity of these methods and models and extending them to processes in which nonsteady effects, sources, and sinks due to phase and chemical transformations and diverse nonlinear phenomena are of fundamental importance [2]. In practice, as before, the phenomenological model based on the concept of parallel transport in the two phases of a heterogeneous medium is most often used; this model leads to a system of two linear equations with constant coefficients [1, 3, 4] or to a single equivalent transport equation, which may be formally obtained from this system [5, 6].

The applicability of these equations is limited to processes which are very close to steady state. Generalization to a situation which is very unsteady is difficult in that

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several relaxation processes with comparable (generally speaking) relaxation times occur simultaneously in the system and dispersional phenomena associated both with transit through the heterogeneous medium and with transfer between its phases are present simultaneously [7]. The dispersion of the interphase flux was considered recently under the assumption that only relaxation of the temperature or concentration field within the particles of disperse phase in practically homogeneous conditions has the determining role [8, 9]. This leads directly to the explanation of a series of generally observed effects. Results of similar significance were obtained recently for polydisperse media, taking account of some nonlinear phenomena at the phase interface [10, 11]. In all cases, nonsteady behavior of the phase-transfer coefficient leads to a nonlocal (in time) equivalent equation containing integral hereditary terms [7-11]. Analogous equations with a series of simplifying assumptions were obtained earlier in filtration theory in cracked porous media, the mathematical formulation of which has much in common with that of the problem of heat and mass transfer [12-15]. In the present work, for the example of heat transfer in a granular medium, in the absence of contact heat conduction through the grain body, the dispersion of both interphase transfer and effective heat flux is taken into account, as well as the inhomogeneity of the temperature field.

2. Bearing in mind that, as in [8, 9], the dynamics of temperature-field variation within an individual particle must be considered, taking account of the temperature relaxation and inhomogeneity outside the particle, the same transfer theory as in [7, 16] is used, on the basis of the method of ensemble averaging and the concepts of self-consistent field theory. According to this theory, the effective heat fluxes in a granular medium and interphase heat transfer are expressed as integrals, the integrands of which depend on the distribution of the mean temperature in a single (trial) particle. It is convenient to introduce dimensionless coordinates r with the scale a and time Fo with the scale a^2/κ_2 and to apply at once Laplace transformation with respect to the time with parameter p. This yields the equations

$$\frac{\varkappa_2}{a^2}\varepsilon c_1 p\tau_1 = -\frac{1}{a}\nabla \mathbf{q} + H, \quad \frac{\varkappa_2}{a^2}\rho c_2 p\tau_2 = -H, \tag{1}$$

where

$$\begin{gathered} \mathbf{q}(\mathbf{R}) = -\frac{\lambda_1}{a}\nabla\tau - (\lambda_2 - \lambda_1)\frac{3\rho}{4\pi a}\int\limits_{|\mathbf{R}-\mathbf{R}'|\leqslant 1}\nabla_{\mathbf{R}}\tau^*(\mathbf{R}|\mathbf{R}')\,d\mathbf{R}', \\ H(\mathbf{R}) = -\frac{3\rho\lambda_2}{4\pi a^2}\int\limits_{|\mathbf{R}-\mathbf{R}'|\leqslant 1}\Delta_{\mathbf{R}}\tau^*(\mathbf{R}|\mathbf{R}')\,d\mathbf{R}', \quad \tau = \varepsilon\tau_1 + \rho\tau_2, \end{gathered} \tag{2}$$

and the integration is taken over the radius vector R' of the trial particle (here and below, the functions and their Laplace transforms are denoted by the same symbols). In addition, the following formulas may be written for q and H

$$\mathbf{q} = -\frac{\lambda_1}{a}\beta\nabla\tau, \quad H = -\frac{\lambda_2}{a^2}\rho p\mu\tau, \tag{3}$$

where β and μ are unknown functions of the parameters of the medium and p; the dependence on p determines the dispersion of the effective thermal conductivity $\lambda_1\beta$ and also the interphase heat transfer.

Using Eq. (3), Eq. (1) is written in the form

$$\begin{gathered} s^2\tau = \Delta\tau, \quad \varepsilon\tau_1 = (1-\rho\mu)\tau, \quad \tau_2 = \mu\tau, \\ s^2 = \frac{\varkappa_2}{\varkappa_1}\left[1 + \rho\left(\frac{c_2}{c_1} - 1\right)\mu\right]\frac{p}{\beta}, \end{gathered} \tag{4}$$

and hence it follows, in particular, that the transforms of the mean temperatures of the phases are expressed in terms of the transform of the mean temperature of the medium as a whole.

Assuming, for the sake of simplicity, that the mean temperatures depend on only a single Cartesian coordinate $z = r\cos\phi$, $r = R - R'$ (it may be shown that this does not limit the generality of the theory), $\tau(R)$ is written as a Taylor expansion

$$\tau(\mathbf{R}) = \tau(\mathbf{R}' + \mathbf{r}) = \tau' + E'z + M'z^2 + \dots,$$

and then, using Eq. (4) and expansion in spherical functions, it is found that

$$\tau(\mathbf{R}) = \tau'\left(1 + \frac{s^2r^2}{6}\right)P_0 + E'rP_1 + \tau'\frac{s^2r^2}{3}P_2 + \dots, \tag{5}$$

where $P_n = P_n(\cos\phi)$ are Legendre polynomials. Analogous expansions may be written relative to the point R, which determines the values of the coefficients in Eq. (5)

$$\tau' = \tau(\mathbf{R} - \mathbf{r}) = \tau\left(1 + \frac{s^2r^2}{6}\right)P_0 - ErP_1 + \tau\frac{s^2r^2}{3}P_2 + \dots,$$
$$E' = EP_0 - \tau s^2 rP_1. \tag{6}$$

The continuum method of describing transport processes in a heterogeneous medium is only possible, in principle, in the case where the linear scale of the mean fields is much greater than the linear scale of the internal structure of the medium. This entails that $s^2 \ll 1$. Since the Laplace-transformation parameter p appears in the definition of s^2 in Eq. (4), this requirement in fact imposes a constraint on the degree of nonsteady behavior of the processes which may be considered within the framework of continuum theory.

To determine the field $\tau^*(R|R')$ appearing in the integrals in Eq. (2), the special problem of a trial particle immersed in some hypothetical medium whose thermal conductivity depends on the distance to the particle surface must be considered [7, 16]. The character of this dependence is determined by the type of packing of the particles; different versions for a monodisperse layer of spheres were considered in [17]. Here, considering more realistic heterogeneous systems, an approximate method of description in which the hypothetical medium consists of a homogeneous continuum with the properties of the heterogeneous medium as a whole in the region outside a sphere of radius $(1 + \chi)a$ concentric with the trial particle and a spherical layer $(1 + \chi)a > r > a$ inside this sphere in which the properties coincide with those for a continuous phase is used. A model of this type was first proposed in [18], in connection with the problem of determining the effective elastic properties of composite materials. The effective thermal conductivity of the disperse layer was determined within the framework of this approach when $\chi = 1$ in [16, 17].

In the given case, the problem of a trial-particle problem takes the following form, taking account of the smallness of s^2

$$\tau^* p = \Delta\tau^*, \quad 0 < r \leqslant 1; \quad \Delta\tau'' = 0, \quad 1 < r \leqslant 1 + \chi; \quad \Delta\tau''' = 0, \quad r > 1 + \chi;$$
$$\tau^* = \tau + \tau'', \quad \lambda_2 \mathbf{n}\nabla\tau^* = \lambda_1\beta\mathbf{n}\nabla\tau + \lambda_1\mathbf{n}\nabla\tau', \quad r = 1;$$
$$\tau'' = \tau''', \quad \lambda_1\mathbf{n}\nabla\tau'' = \lambda_1\beta\mathbf{n}\nabla\tau''', \quad r = 1 + \chi;$$
$$\tau^* < \infty, \quad r = 0; \quad \tau''' \to 0, \quad r \to \infty. \tag{7}$$

Note that the boundary condition of continuity of the heat flux in this problem differs significantly from the conditions specified within the framework of the phenomenological models in which the mean heat flux is expressed as $\lambda_1\beta\nabla\tau$ not only far from the trial particle but also in the surface layer [16].

At the level of accuracy corresponding to taking account of only the first two terms in the expansions in Eqs. (5) and (6), the solution of Eq. (7) is obtained in the form of a sum of elements proportional to P_0 and P_1. In particular, the following expression may be written for τ^*

$$\tau^* = [\tau'\alpha_0 I_{\frac{1}{2}}(y)P_0 + E'\alpha_1 I_{\frac{3}{2}}(y)P_1]\,y^{-1/2}, \tag{8}$$

where

$$\alpha_0 = \sqrt{\frac{\pi}{2}}\,\frac{\sqrt{p}}{\operatorname{sh}\sqrt{p}}\left[1 + \frac{\varkappa p(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}, \quad F = 3\,\frac{\sqrt{p}\operatorname{cth}\sqrt{p} - 1}{p},$$
$$\alpha_1 = \frac{3\sqrt{\pi/2}\,[(2\beta^2+2)(D-1) + \beta(5D+4)]/\operatorname{sh}\sqrt{p}}{[2D(2\beta+1) - 2 + 2\beta]F + \varkappa(3-2F)[D(2\beta+1) + 2 - 2\beta]},$$
$$y = r\sqrt{p}, \quad \varkappa = \lambda_2/\lambda_1, \quad D = (1+\chi)^3. \tag{9}$$

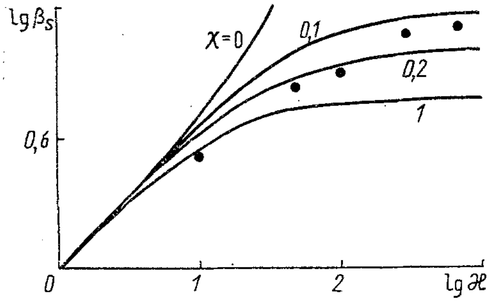

Fig. 1. Dependence of the dimensionless effective thermal conductivity of the system in steady conditions on κ when ρ = 0.6 for various χ (given on the curves); the points correspond to the experimental data of [20].

Integration in Eq. (2) in accordance with Eqs. (8) and (9) gives

$$\int\limits_{|\mathbf{R}-\mathbf{R}'|\leq 1} \nabla_{\mathbf{R}}\tau^*(\mathbf{R}|\mathbf{R}')\,d\mathbf{R}' = \frac{4}{3}\pi E\left\{(F-1)\left[1+\frac{\varkappa p(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}+\right.$$
$$\left.+\frac{(2\beta^2+2)(D-1)+\beta(5D+4)}{2D(2\beta+1)-2+2\beta+\varkappa(3/F-2)[D(2\beta+1)+2-2\beta]}\right\} \quad (10)$$
$$\int\limits_{|\mathbf{R}-\mathbf{R}'|\leq 1} \Delta_{\mathbf{R}}\tau^*(\mathbf{R}|\mathbf{R}')\,d\mathbf{R}' = \frac{4}{3}\pi p\tau F\left[1+\frac{\varkappa p(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}.$$

Comparison of Eqs. (2) and (10) with Eq. (3) gives the transcendental equations

$$\beta = 1+(\varkappa-1)\rho\left\{(F-1)\left[1+\frac{\varkappa p(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}+\right.$$
$$\left.+\frac{(2\beta^2+2)(D-1)+\beta(5D+4)}{2D(2\beta+1)-2+2\beta+\varkappa(3/F-2)[D(2\beta+1)+2-2\beta]}\right\}, \quad (11)$$
$$\mu = F\left[1+\frac{\varkappa p(1+\beta\chi)}{3\beta(1+\chi)}F\right]^{-1}$$

for the unknowns β and μ, which permits closure of the theory.

Thus, Eq. (11) completely determines the relation in Eq. (4) between the transforms of the mean temperatures of the phase and the medium as a whole; the latter may be determined from the solution of the corresponding boundary problem for Eq. (4). If weakly nonsteady processes are considered, i.e., sufficiently small p is assumed, approximate relations for β and μ in the form of a sum of terms proportional to the powers of p may be obtained from Eq. (11), and an equation for any of the mean temperatures from Eq. (4). Its inversion leads at once to an integro-differential nonlocal (in time) equation of the same type as in [7-15]. However, in the general case, as noted in [9], it is expedient to solve the problem in transforms, with inversion only in the final stage, for which there are well-developed numerical methods [19].

3. Consider the asymptotic versions of this theory. First of all, in almost-steady processes $p \ll 1$, $F \simeq 1$, it follows from Eq. (11), neglecting terms of order p and above, that

$$p \approx \beta_s = 1+\frac{\rho(\varkappa-1)[(2\beta^2+2)(D-1)+\beta(5D+4)]}{2\beta[D(2+\varkappa)+1-\varkappa]+D(2+\varkappa)+2(\varkappa-1)},\ \mu\approx\mu_s = 1. \quad (12)$$

The second relation simply states the approximate equality of the temperatures of the phases in weakly nonsteady processes. The first relation in Eq. (12) determines the effective steady thermal conductivity $\lambda_* = -\lambda_1\beta_s$ with arbitrary χ; when χ = 0 and 1, this quantity was calculated in [7, 16]. In the general case, χ must be regarded as some semiempirical parameter which globally characterizes the properties of the particles of a granular medium and their packing or, in an even more general case, the structural properties of a nongranular heterogeneous material. Even for an ideal layer with identical spherical parti-

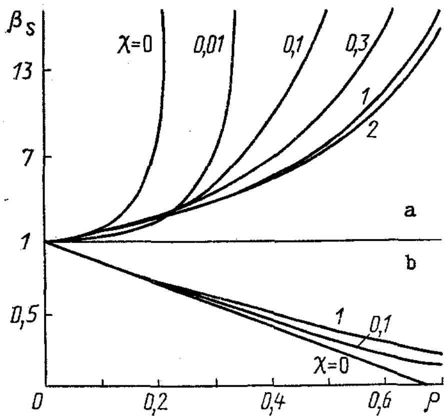

Fig. 2. Dependence of the dimensionless steady effective thermal conductivity on ρ for various κ (given on the curves); $\kappa \to \infty$ (a) and $\kappa = 0$ (b).

cles, the value $\chi = 1$ following from general theory under a series of assumptions leads to values of the effective thermal conductivity that are somewhat too low [16, 17], i.e., in this case, it is expedient to choose χ on the basis of the results of comparing theory and experiment. As an example, comparison with the data of [20] for a granular layer with $\rho = 0.6$ is shown in Fig. 1 (only some reference points reflecting the general trend are shown). Hence it follows that $\chi \simeq 0.2$.

The dependence of $\beta_S = \lambda_*/\lambda_1$ on ρ for the limiting situations when $\kappa = \lambda_2/\lambda_1 \to \infty$ and $\kappa = 0$ at various χ is shown in Fig. 2. These curves may be useful in estimating the effective thermal conductivity of broad classes of heterogeneous media, if specific values of χ are determined from a few experiments. Whereas χ is around 0.2 for dense granular media, as follows from an analysis of Fig. 1, χ may easily reach or even exceed unity for loosely packed layers of particles of complex, significantly nonspherical, form. Conversely, it may be expected that the characteristic values of χ in heterogeneous materials of the type of cracked porous media will be close to zero.

Terms of first order in Eq. (11) are now taken into account in Eq. (11). The first relation in Eq. (12) is unchanged here, but the second is replaced by

$$\mu \approx \left[1 + \frac{\varkappa p(1+\beta_s\chi)}{3\beta_s(1+\chi)}\right]. \tag{13}$$

In this case, it follows from Eq. (4) that

$$\begin{gathered}\tau_2 = \frac{\varepsilon}{(\mu^{-1}-1)}(\tau_1-\tau_2) = \frac{3\beta_s\varepsilon(1+\chi)}{\varkappa p(1+\beta_s\chi)}(\tau_1-\tau_2),\\ \lambda_1\beta_s\Delta\tau_1 = \varepsilon\left[\varkappa_2 c_1 p\tau_1 + 3\rho\lambda_1\frac{\beta_s(1+\chi)}{(1+\beta_s\chi)}(\tau_1-\tau_2)\right] + \\ + \frac{\varkappa\rho p^2\varkappa_2(1+\beta_s\chi)}{3\beta_s(1+\chi)}\left[c_1\tau_1 + c_2\tau_2\frac{\rho}{\varepsilon}\right].\end{gathered} \tag{14}$$

Again neglecting terms of order p^2, performing inverse Laplace transformation, and returning to the dimensional variables x and t, the following equations are obtained

$$\begin{gathered}\varepsilon c_1\frac{\partial\tau_1}{\partial t} = \lambda_*\Delta\tau_1 - \frac{\lambda_1}{4a^2}A^2(\tau_1-\tau_2),\quad \lambda_* = \lambda_1\beta_s,\\ \rho c_2\frac{\partial\tau_2}{\partial t} = \frac{\lambda_1}{4a^2}A^2(\tau_1-\tau_2),\quad A^2 = 12\rho\varepsilon\frac{(1+\chi)}{(1+\beta_s\chi)}\beta_s,\end{gathered} \tag{15}$$

which coincide with the equations of heterogeneous transfer through the phases of the medium; these equations are very often used in practical calculations. The parameter A was first introduced in [1, 3, 4], where it was assumed to be two. In fact, it is a function both of ρ and χ and, through β_S, also of κ. Characteristic curves of A as a function of χ, i.e., in fact, of the structural features of the medium when $\rho = 0.6$ and for various κ, are shown

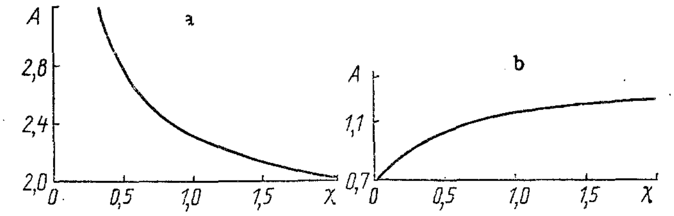

Fig. 3. Parameter A as a function of χ when $\rho = 0.6$ and $\kappa \to \infty$ (a), $\kappa = 0$ (b).

in Fig. 3, from which it is evident that A = 2 may only be regarded as some mean value of the true dependences.

In the absence of any nonphenomenological procedure for deriving Eq. (15), the above conclusion is evidently of definite methodological interest. Even more importantly, the conditions of applicability of this system, which have previously been repeatedly (and on the whole unsuccessfully) discussed, are now clear.

For small times ($p \gg 1$, $F \simeq 3p^{-1/2}$), it follows from Eq. (11) that (the requirement $p \gg 1$ may be consistent with the condition $s^2 \ll 1$)

$$\beta \approx 1, \quad \mu \approx 0. \tag{16}$$

This corresponds to heat transfer practically only through the continuous phase at small times. The particles still are not able to absorb a pronounced quantity of heat and so have practically no influence on the transfer. Note that Eq. (15) formally gives the correct asymptote for small times if $\lambda_* = \lambda_1$ is assumed.

4. To illustrate various relaxational and dispersional effects influencing the character of development of the process, the features of the heating of an initially cold granular layer from a solid wall with a constant temperature T_W is considered, as in [8, 9], within the framework of the first boundary problem. The solution of Eq. (4) for the transform of the mean layer temperature is

$$\tau = (T_w/p)\exp(-sz),$$

and the transform of the heat-flux to the layer is

$$Q = \frac{\lambda_1 T_w}{a}\left\{\left(\frac{\varkappa_2\beta}{\varkappa_1 p}\right)\left[1 + \left(\frac{c_2}{c_1} - 1\right)\rho\mu\right]\right\}^{1/2}. \tag{17}$$

As $p \to 0$ ($\mathrm{Fo} \to \infty$), Eq. (17) takes the form

$$Q_s = \frac{W}{\sqrt{p}}, \quad W = \frac{\lambda_1 T_w}{a}\left\{\frac{\varkappa_2\beta_s}{\varkappa_1}\left[1 + \left(\frac{c_2}{c_1} - 1\right)\rho\right]\right\}^{1/2}. \tag{18}$$

The Nusselt number is defined as

$$\mathrm{Nu} = \frac{Q}{W} = \frac{1}{\sqrt{p}}\left\{\frac{\beta}{\beta_s}\left[\frac{1 + (c_2/c_1 - 1)\rho\mu}{1 + (c_2/c_1 - 1)\rho}\right]\right\}^{1/2}. \tag{19}$$

The Nusselt number is defined analogously for the solution of the same problem in [9], where only the relaxation process inside the particles is taken into account

$$\mathrm{Nu} = \frac{1}{\sqrt{p}}\left[\frac{1 + kF}{1 + k}\right]^{1/2}, \quad k = \frac{c_2\rho}{c_1\varepsilon}. \tag{20}$$

For comparison, the expression for a homogeneous mass with thermal conductivity $\lambda_* = \lambda_1\beta_S$ is also given here

$$\mathrm{Nu} = p^{-1/2} \doteqdot 1/(\pi\,\mathrm{Fo}) \tag{21}$$

as well as the expression for Nu obtained from the solution of Eq. (15)

$$\mathrm{Nu} = \frac{1}{\sqrt{p}}\left[\frac{1 + k + k_1 p}{(1 + k)(1 + k_1 p)}\right]^{1/2}, \quad k_1 = \frac{4\rho\varkappa}{A^2}, \tag{22}$$

where k is defined in Eq. (20).

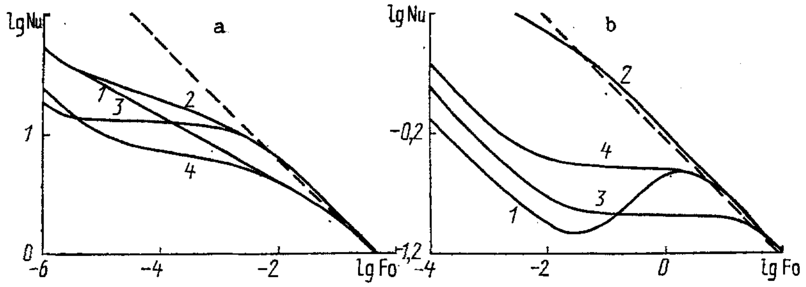

Fig. 4. Dependences Nu(Fo) for $\rho = 0.6$; $c_2/c_1 = 10^3$; $\kappa = 0.1$; $\chi = 0.07$ (a); $\kappa = 100$; $\chi = 0.2$ (b); the dashed curves correspond to the solution in Eq. (21) for the usual heat-conduction equation; curves 1 and 2 to Eqs. (19) and (20), respectively; curve 3 to the solution in Eq. (22) of Eq. (15) with A = 2; and curve 4 to Eq. (19) but with $\beta = \beta_S$, which corresponds to taking no account of the dispersion of β and retaining only one term in the expansions in Eqs. (5) and (6).

The dependences Nu(Fo) obtained from Eqs. (19)-(22) by numerical inversion of the Laplace transformation for various κ and χ are compared in Fig. 4 [19]. Analysis of the curves leads to the following conclusions.

Taking account of the dispersion of the effective thermal conductivity $\lambda_1\beta$ leads to the appearance of a minimum on the curve of Nu(Fo). This minimum appears if the thermal conductivity of the particles exceeds that of the continuous phase (Fig. 4b); it becomes more strongly expressed as the ratio $\varkappa = \lambda_2/\lambda_1$ increases. With increase (decrease) in κ, the range of times at which relaxational phenomena appear is shifted to larger (smaller) Fo. The presence of a minimum on curve 1 (Fig. 4b) may be interpreted as follows. At small times after the onset of the process, heat transfer occurs only through the continuous phase; tortuosity effects are unable to appear and the effective thermal conductivity of the medium is simply the thermal conductivity of the continuous phase. Then, heat absorption by the particles begins, and the drop in the heat flux from the wall stops; the presence of such a "shallow" intermediate asymptote was noted in [8]. At large thermal conductivity of the particles, this absorption may be very considerable and leads to sharp decrease in temperature of the continuous phase (the heat is absorbed by the particles more rapidly than it is supplied from the wall), and hence to increase in temperature gradient, which, in turn, results in increase in heat flux from the wall. This leads to the appearance of a minimum on the curve of Nu(Fo). Subsequently, heat-transfer processes between the disperse and continuous phases are completed, their temperatures equalize, and the granular medium behaves as a homogeneous medium with $\lambda_* = \lambda_1\beta_S$.

The equation obtained in [9] to describe heterogeneous transfer (curve 2 in Fig. 4a, b) is only valid at small κ ($\lambda_2 \ll \lambda_1$), as a consequence of the assumption that the temperature field in the continuous phase is homogeneous at scales of the order of the particle dimension. This also applies to the result obtained in [12, 13], and was mentioned in [15]. Note that this assumption may be valid, for example, for problems of liquid filtration in cracked porous media, where the conductivity through the cracks may significantly exceed the conductivity through the porous blocks. However, for example, for heat transfer in granular media with $\kappa > 1$, for example, this assumption is invalid and, as shown in Fig. 4, leads to qualitative disagreement with the solution in Eq. (19). For small κ, in a granular medium, heating of a large number of particles begins practically at once (as a result of fast heat transfer through the continuous phase). Relaxational effects in this case are more weakly expressed (Fig. 4a).

The solution of the simple system in Eq. (15) when $\kappa \lesssim 10$ describes the dynamics of heat-transfer processes in a granular layer qualitatively correctly on the whole, practically coinciding with curve 4 in Fig. 4. At small Fo, it is found to behave much more correctly than the solution of the equivalent elliptical equation, which tends rapidly to infinity when Fo < 10. This behavior of Eq. (15) is probably due to the presence of correct asymptotes both as $Fo \to \infty$ and as $Fo \to 0$ in the system. In this respect, the solutions of Eq. (15) may evidently be regarded as reasonable approximations in the sense of [21].

Finally, the use of solutions of Eq. (15) may be recommended for practical engineering calculations in situations where $\kappa \lesssim 10$ when modeling heat and mass transfer in heterogeneous media. For cases where $\kappa > 10$, however, the failure to take account of dispersional effects leads to qualitatively incorrect description of the processes. In these cases, the equations from the present work must be used.

The above results on modeling heterogeneous transfer for thermal problems are probably more methodological in character than a basis for engineering calculations, as a result of the small time in which relaxational effects appear. For example, for a granular layer of glass balls (radius 1 mm), the dimensionless time $Fo = 10^{-2}$ corresponds to $t \sim 10^{-4}$ sec. It is difficult to measure such times in experiments, and they are rarely of practical importance. At the same time, these results may be of great practical value for filtration problems in inhomogeneous porous media. Thus, for filtration in a cracked porous medium, with a block size of 1 m and a piezoconductivity of 10^{-4} m²/sec, $Fo = 10^{-2}$ corresponds to $t \sim 100$ sec, which may be measured experimentally. In a number of cases, the characteristic times may reach tens of minutes and be of significant practical value, for example, in the case of the processes considered in [13]. Note that the inequality $\kappa_2 \ll \kappa_1$, which is fundamental to the theory, may easily be satisfied for materials of the type of cracked porous media, when the conduction of the porous blocks is much less than that of the surrounding continuous phase modeling the system of cracks.

In conclusion, promising lines of development of the theory may be briefly noted. Above all, in many situations, conductivity through the body of the disperse-phase particles is significant. This may be the case both in heat-transfer problems [22, 23] and in the filtration of liquids in cracked porous media [6]. In many mass-transfer processes, nonlinear effects associated with the behavior of impurity at the phase interface play an important role [10, 11]. Finally, in a series of problems, account must be taken of phase and chemical transformations at this surface, sometimes accompanied by change in particle size of the disperse phase [2]. In filtration problems and also in problems of high-temperature heat transfer in the presence of radiant heat conduction, the nonlinear dependences of the permeability or thermal conductivity on the pressure or temperature are also important. All these problems of significant practical value may, in principle, be considered on the basis of methods analogous to those above.

NOTATION

A, parameter in Eq. (15); $D = (1 + \chi)^3$; a, particle radius; c, specific heat per unit volume; E, M, coefficients of the expansion; F, function of the parameter p defined in Eq. (9); Fo, dimensionless time; H, functional defined in Eq. (1); k_1, k_2, parameters in Eqs. (20) and (22); Nu, Nusselt number; p, Laplace-transformation parameter; Q, heat flux to granular layer from wall; q, heat flux in Eq. (1); R, R', r, r, z, dimensionless spatial coordinates; s, coefficient defined in Eq. (4); T_W, wall temperature; t, time; W, coefficient in Eq. (18); x, dimensional spatial coordinates; y, parameter in Eq. (9); α_i, coefficients in Eqs. (8) and (9); β, μ, functions of p introduced in Eq. (3); ε, volume fraction of continuous phase; κ_i, thermal diffusivity; $\kappa = \lambda_2/\lambda_1$; λ, thermal conductivity; λ_*, effective thermal conductivity; $\rho = 1 - \varepsilon$; τ_i, τ, mean temperatures of the phases and the medium as a whole; τ_*, temperature inside trial particle; τ'', τ''', perturbations of mean temperature; ϕ, polar angle; χ, dimensionless thickness of continuous-phase layer around trial particle. Indices: 1, 2, continuous and disperse phases, respectively; s, steady state; a prime denotes quantities calculated for the center of the trial particle.

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